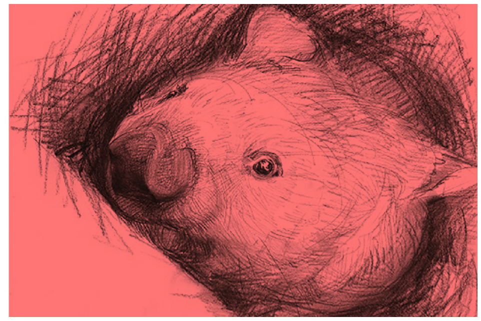
# WOMBAT 2021

## Online Edition

Workshop on Optimisation, Metric Bounds, Approximation and Transversality

13 — 17 December 2021



# Program and Abstracts

Compiled on 17th December 2021 3:30am GMT.

The Sixth Workshop on Optimisation, Metric Bounds, Approximation and Transversality (WOMBAT2021) will be online 13–17 December 2021.

## Website:

https://wombat.mocao.org

## Discussion Channel:

https://discord.gg/JETRZPYf3s

## Childcare Support:

WOMBAT2021 has received AustMS-WIMSIG Anne Penfold Street Award to support the arrangement of childcare for the participants. If you require such support, please contact Vera Roshchina.

## Organisers:

| | |
|---|---|
| Dr Nadia Sukhorukova | Swinburne University of Technology |
| Dr Minh Dao | Federation University |
| Dr Reinier Diaz Millan | Deakin University |
| Prof Andrew Eberhard | RMIT University |
| Dr Nam Ho-Nguyen | The University of Sydney |
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## Sponsors:

Mathematics of Computation and Optimisation (MoCaO) Special Interest Group of the Australian Mathematical Society (AustMS)

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# Workshop Program

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| 13:00–14:00 | **Javier Peña** (Keynote discussion) |
| 14:00–14:30 | **David H. Gutman**<br>Coordinate Descent Without Coordinates: Tangent Subspace Descent on Riemannian Manifolds |
| 14:30–15:00 | **Vinesha Peiris**<br>Rational activation functions in neural networks with uniform based loss functions and its applications in classification |
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# Abstracts

## Directional differentiability, coexhausters, codifferentials and polyhedral DC functions

**Majid Abbasov**
Saint Petersburg State University; Institute for Problems in Mechanical Engineering of the Russian Academy of Sciences
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Codifferentials and coexhausters are used to describe nonhomogeneous approximations of a nonsmooth function. Despite the fact that coexhausters are modern generalizations of codifferentials, the theories of these two concepts continue to develop simultaneously. Moreover, codifferentials and coexhausters are strongly connected with DC functions. We trace analogies between all these objects, and prove the equivalence of the boundedness and optimality conditions described in terms of these notions. This allows one to extend the results derived in terms of one object to the problems stated via the other one. We also study of connection between nonhomogeneous approximations and directional derivatives and formulate optimality conditions in terms of nonhomogeneous approximations.

## Cutting planes method for solving binary nonlinear problems

**Hoa Thi Bui**
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In this talk, we discuss a general approach based on cutting planes for solving nonlinear (possibly non-convex) binary optimisation problems. The cutting plane method here can be viewed as the discrete analogue of bundle methods. We provide a convergence analysis that yields an upper bound on the number of iterations required. Moreover, using tools from variational analysis, we provide necessary and sufficient dual optimality conditions. As a by-product of these optimality conditions, we show that if the objective function is quasiconvex and the feasible set is defined only with linear constraints, once the optimal solution is first added, the cutting plane method will converge in the next iteration.

## Generalized Bregman distances

**Regina S. Burachik**
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Bregman distances are fundamental tools in many areas: feasibility problems, generalized proximal point methods, generalized projections, probability theory, and many more. The usual way of constructing a Bregman distance is by means of a specific convex function. We look at these distances from the point of view of maximally monotone operators, and show that they can be obtained as a particular case of a general family of distances associated to a maximally monotone operator, when the maximally monotone map is the subdifferential of the convex function. Based on this idea, we extend the notion to a distance between two point to set maps, where only one of them is maximally monotone. We show that these new distances, which we call Generalized Bregman distances, can be used to identify solutions of variational problems, such as variational inequalities and DC problems. This talk comprises joint work from 2018 (with Juan Enrique Martínez-Legaz), until 2021 (with Minh N. Dao and Scott B. Lindstrom).

**Tutorial: Survey on enlargements, properties, applications and examples**

In this tutorial, I will recall the definition of enlargement of a maximally monotone operator, and show how the connection with a family of convex functions is achieved. Then I will recall key properties of the family of enlargements and the family of convex functions, called the Fitzpatrick function. I will also mention some applications of these concepts. Time permitting, I will illustrate with some specific examples of enlargements and associated functions.

## Ranges of non-smooth mappings

**Radek Cibulka**
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We derive conditions ensuring that the range of a given, in general set-valued, mapping with a closed graph and a compact convex domain covers a prescribed set. We focus on approximations determined by a set of bounded linear mappings. We demonstrate that our approach is highly flexible and provides the unified treatment of various, in general non-local, covering properties of non-smooth mappings such as constrained directional semiregularity, metric regularity, and strong metric regularity. The conditions presented cover, unify, and extend such well-known results as Pourciau's open mapping theorem and Clarke's inverse and implicit function theorems as well as their generalizations and variants established, for example, by V. Jeyakumar and D.T. Luc by using upper semi-continuous unbounded pseudo-Jacobians; by A. Neumaier by considering various interval extensions of the derivative of a smooth mapping; by H. Halkin by using the notion of a screen; and by T.H. Sweetser by the notion of a shield. Finally, we provide conditions guaranteeing that the non-linear image of a compact convex set contains a prescribed ordered interval which has direct applications in power network security management such as preventing the electricity blackout.

## The intrinsic core and minimal faces of convex sets in general vector spaces

**Reinier Diaz Millan**
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Intrinsic core generalises the finite-dimensional notion of the relative interior to arbitrary (real) vector spaces. Our main goal is to provide a self-contained overview of the key results pertaining to the intrinsic core and to elucidate the relations between intrinsic core and facial structure of convex sets in this general context.

## Bilevel Programming: A Personal Journey

**Joydeep Dutta**
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I have been involved with bilevel programming after meeting Stephan Dempe in 2004. In this talk I discuss the problems of bilevel programming that I have thought about and how I along with several collaborators tried to answer them. Apart from trying to understand the relation between optimistic solutions and the solutions of a slightly modified bilevel programming one of my most enjoyable work was to actually show that bilevel programming is not a special case of mathematical programming under complementarity constraints(MPCC). The idea that bilevel problems can be solved by replacing the lower level problem with it's KKT condition and thus converting it to a single problem which was an MPCC and ten solve it really did not work even if the lower-level was convex. Apart from that I had been deeply interested to devise necessary conditions for optimality for bilevel problems with convex lower level problems. Due to the nested nature of the problem it is a challenging task. I this talk I also highlight how I along with my collaborators and also other researchers have tried to tackle this challenge.

**Tutorial: Simple Bilevel Programming and Extensions: A Tutorial**

My current research focusses on simple bilevel programming which the problem of minimizing a convex function over the solution set of another convex optimization problem. Though the problem appears to be simple any attempt to write down the optimality conditions brings in all the difficulties associated withe the original bilevel problems in the leader-follower framework. We show step ny step where such kind of problem arises in a natural way and how algorithms can be developed to tackle such problems and this includes mu current on going work which would be outlined in the talk so as to motivate young graduate students to take part in this and emerging aspect of bilevel optimization.

## Monotone Operators and Closures Operations for Convex Sets in $X \times X^*$

**Andrew Eberhard**
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In the study of monotone operators $M : X \to X^*$ is greatly facilitated via the used of convex function defined on the product of Banach spaces $X \times X^*$ that are minorised by the duality product and make contact there along the graph of the monotone operator. The closure that such representative functions naturally exhibits is that arising in the construction of the Fitzpatrick function and leads us to study $s \times w^*$-closure operations as applied to convex sets in $X \times X^*$. In this talk we discuss some of the inherent topological difficulties involved and how these issue do not arise when $X$ is reflexive or the monotone operator is of dense type. We are lead to discuss a variant of the Banach–Dieudonné theorem as it applies to convex sets in $X \times X^*$ and a new way of studying such closures for convex sets. We ask whether these closures can be sequentially characterised for convex sets and find a super set of reflexive spaces for which this is the case.

## Stochastic incremental mirror descent algorithms with Nesterov smoothing

**Sorin-Mihai Grad**
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We propose a stochastic incremental mirror descent method constructed by means of the Nesterov smoothing for minimizing a sum of finitely many proper, convex and lower semicontinuous functions over a nonempty closed convex set in a Euclidean space. The algorithm can be adapted in order to minimize (in the same setting) a sum of finitely many proper, convex and lower semicontinuous functions composed with linear operators. Another modification of the scheme leads to a stochastic incremental mirror descent Bregman-proximal scheme with Nesterov smoothing for minimizing the sum of finitely many proper, convex and lower semicontinuous functions with a prox-friendly proper, convex and lower semicontinuous function in the same framework. Different to the previous contributions from the literature on mirror descent methods for minimizing sums of functions, we do not require these to be (Lipschitz) continuous or differentiable. Applications in Logistics, Tomography and Machine Learning modelled as optimization problems illustrate the theoretical achievements. The talk is based on joint work with Sandy Bitterlich.

## Coordinate Descent Without Coordinates: Tangent Subspace Descent on Riemannian Manifolds

David H. Gutman
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We extend coordinate descent to manifold domains, and provide convergence analyses for geodesically convex and nonconvex smooth objective functions. Our key insight is to draw an analogy between coordinate blocks in Euclidean space and tangent subspaces of a manifold. Hence, our method is called tangent subspace descent (TSD). The core principle behind ensuring convergence of TSD is the appropriate choice of subspace at each iteration. To this end, we propose two novel conditions, the $(C, r)$-norm and $C$-randomized norm conditions on deterministic and randomized modes of subspace selection respectively, that promise convergence for smooth functions and that are satisfied in practical contexts. We propose two subspace selection rules, one deterministic and another randomized, of particular practical interest on the Stiefel manifold. Our proof-of-concept numerical experiments on the Sparse Principal Component Analysis problem demonstrate TSD's efficacy.

## Dual estimates of nonconvexity of sets in Hilbert spaces

Abderrahim Hantoute
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Dual estimates of non-convexity are provided for sets in Hilbert spaces, using the size of the associated projections and weak projections. Such estimates give rise to a precise relationship between the Hausdorff excess to a set from its convex hull, the associated effective deviation standard function, and the deviation in the projection mapping which can result from projecting simultaneously onto the set and its closed convex envelope. The amount of non-convexity is then shown to be proportional to the size of the projections on the set. Accordingly, we give a new quantified version of the Bunt-Klee theorem on the convexity of Chebychev sets.

## Accelerating convergence of a globalized sequential quadratic programming method to critical Lagrange multipliers

**Alexey F. Izmailov**
Lomonosov Moscow State University, Russia
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This work concerns the issue of asymptotic acceptance of the true Hessian and the full step by the sequential quadratic programming algorithm for equality-constrained optimization problems. In order to enforce global convergence, the algorithm is equipped with a standard Armijo linesearch procedure for a nonsmooth exact penalty function. The specificity of considerations here is that the standard assumptions for local superlinear convergence of the method may be violated. The analysis focuses on the case when there exist critical Lagrange multipliers, and does not require regularity assumptions on the constraints or satisfaction of second-order sufficient optimality conditions. The results provide a basis for application of known acceleration techniques, such as extrapolation, and allow the formulation of algorithms that can outperform the standard SQP with BFGS approximations of the Hessian on problems with degenerate constraints. This claim is confirmed by some numerical experiments.

## Multiple Path Planning for Maximum Information

**Yalçin Kaya**
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We consider the problem of finding optimal paths of multiple observers, or sensors, moving at constant speeds, to estimate the position of a stationary target/emitter/beacon, using only bearing angle measurements. The generated paths are optimal in the sense that, along the paths, information, and thus the efficiency of a potential estimator employed is maximized. In other words, the observer paths are collectively deemed optimal if they maximize information so that the location of the target is estimated with smallest uncertainty, in some sense. We formulate and model the process as an optimal control problem maximizing the determinant of the Fisher information matrix, which is one of the possible measures of information. These

kinds of problems are encountered in search-and-rescue operations, the monitoring of animal populations, and defence. We carry out numerical experiments and discuss the optimal solutions obtained. We verify graphically that the necessary conditions of optimality are verified by the numerical solutions. This is an extension of the paper at https://arxiv.org/pdf/2103.02059.pdf from a single observer to multiple observers.

## Characterizing the upper set relation by general functionals

**Elisabeth Köbis**
Norwegian University of Science and Technology
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In this talk, we investigate very general scalarization concepts for solving set-valued optimization problems where the pre-order is induced by the upper set-less relation as introduced by Kuroiwa. The scalarization functionals that we consider in the analysis are not given explicitly, but are rather general functions that satisfy certain properties such as monotonicity, separation, transitivity, translation invariance and transfer of inclusion. Underlined by several examples, we will show how certain combinations of such properties can be used to characterize the set relation by simple inequalities.

## Optimality Conditions without Lipschitzness

**Alexander Y. Kruger**
Centre for Informatics and Applied Optimization, Federation University Australia, Ballarat
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Approximate necessary optimality conditions in terms of Fréchet subgradients and normals for a general optimization problem with a potentially non-Lipschitzian objective function are discussed. The main tools are Ekeland variational principle, the fuzzy Fréchet subdifferential sum rule, and a novel notion of lower semicontinuity relative to a set.

References

Kruger, A. Y. and Mehlitz, P.: Optimality conditions, approximate stationarity, and applications – a story beyond Lipschitzness. arXiv 2110.07268 (2021).

The research is supported by the Australian Research Council, project DP160100854 and the DFG Grant WA 3636/4-2.

## Abadie condition for systems of infinite number of inequalities and equations in Banach spaces under Relaxed Constant Rank Condition

**Krzysztof Leśniewski**
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We consider systems of infinite number of inequalities and equations given by continuously differentiable functions defined on Banach spaces with values in Hilbert spaces. We are representing these systems with the help of coefficients, in a given Schauder basis. We prove that Abadie condition (linearized cone is contained in tangent cone) holds. The main tools are: new infinite-dimensional Relaxed Constant Rank Condition (inspired by the finite-dimensional idea of Minchenko and Stakhovski), Rank Theorem and Ljusternik Theorem.

## Error bounds, facial residual functions and applications to the exponential cone

**Scott B. Lindstrom**
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In this talk, I'll provide the resolution to an open problem I shared at Wombat 2019. We construct a general framework for deriving error bounds for conic feasibility problems. In particular, our approach allows one to work with cones that fail to be amenable or even to have computable projections, two previously challenging barriers. For the purpose, we first show how error bounds may be constructed using objects called facial residual functions. Then, we develop several tools to compute facial residual functions even in the absence of closed form expressions for the projections onto the cones. We demonstrate the use and power of our results by computing tight error bounds for the exponential cone feasibility problem. Interestingly, we discover a natural example for which the tightest error bound is related to the Boltzmann-Shannon entropy. We were also able to produce an example of sets for which a Holderian error bound holds but the supremum of the set of admissible exponents is not itself an admissible exponent.

This talk is about joint work with Bruno F. Lourenco and Ting Kei Pong, from the paper of the same name.

## Subdifferential of the supremum function: walking between continuous and non-continuous settings

**Marco López**
University of Alicante
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In this talk we present general formulas for the subdifferential of the pointwise supremum of convex functions, which cover and unify both the compact continuous and the non-compact non-continuous settings. From the non-continuous to the continuous setting, we proceed by a compactification-based approach which leads us to problems having compact index sets and upper semi-continuously indexed mappings, giving rise to new characterizations of the subdifferential of the supremum by means of the new regularized functions and the enlarged compact index set. In the opposite direction, we rewrite the subdifferential of these new regularized functions by using the original data, also leading us to new results on the subdifferential of the supremum. We shall conclude by giving a couple of applications, the first one concerning the nonconvex Fenchel duality, and the second one establishing Fritz-John and KKT conditions in convex semi- infinite programming.

## On consistent error bounds and convergence rates

**Bruno F. Lourenço**
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In this talk, we discuss the framework of "consistent error bound functions", which can be used to analyze error bounds for convex feasibility problems. Our main result is that the convergence rate of several projection algorithms can be expressed explicitly in terms of the underlying consistent error bound function. In particular, this makes it possible to reason about the behaviour of certain algorithms even in situations where no Holderian error bound is expected to hold. This is a joint work with Tianxiang Liu (Tokyo Tech).

## Structured Nonconvex Optimization: Local and Global Analysis

**Russell Luke**
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We present a global and local analysis for convergence of of a noneuclidean proximal gradient algorithm for minimizing a nonconvex, composite objective function with quadratic structure. This model captures a large majority of relaxed nonconvex quadratic programming problems and is a test case for our broader goal of providing a complete analysis of algorithms for structured nonconvex programming - from local convergence to stationary points, to global convergence, and to identification of statinoary points with local optima.

**Tutorial: Is my algorithm an almost $\alpha$-firmly nonexpansive fixed point iteration?**

I present some examples of how to reformulate common algorithms as fixed point iterations (in linear and metric spaces), and then how to apply the calculus for almost $\alpha$-firmly nonexpansive mappings to quantify the convergence of the algorithm.

## Subdifferentiation of sparsity-promoting functions on Lebesgue spaces

Patrick Mehlitz
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Sparsity-promoting terms are incorporated into the objective function of optimal control problems in order to ensure that optimal controls vanish on large parts of the underlying domain. Typical candidates for those terms are integral functions on Lebesgue spaces based on nonconvex as well as non-Lipschitz integrands of special type and, thus, variationally challenging. In the talk, exact formulas for the Fréchet, limiting, and singular subdifferential of these functions are established. Furthermore, based on a novel theory of approximate stationarity for non-Lipschitz optimization problems, necessary optimality conditions for optimal control problems involving such sparsity-promoting terms are derived.

The talk is based on joint ongoing work with Alexander Y. Kruger and Gerd Wachsmuth.

## Fixed-Time Gradient Dynamics with Tunable Time-Varying Coefficients for Optimisation Problems

Lien Nguyen
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Dynamical systems approach to address the continuous-time optimisation problems has attracted much attention from very early days and become a growing interdisciplinary research area. In this work, we propose fixed-time gradient dynamical systems with time-varying coefficients for continuous-time optimisation. We first investigate the Lyapunov stability conditions that allow us to achieve fixed-time stability of the time-varying dynamical systems. We then use them to deal with continuous-time optimisation problems. We show that under the proposed fixed-time gradient dynamics and by choosing time-varying coefficients, the searching trajectories converge to their optima in fixed-time from any initial points with a very fast rate.

## On the strong subregularity of the optimality mapping in mathematical programming and calculus of variations

**Nikolai Osmolovskii**

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The paper presents sufficient conditions for a strong metric subregularity (SMSr) property of the optimality mapping associated with the Pontryagin local maximum principle for a Mayer's type optimal control problem with general initial/terminal constraints for the state variable and unconstrained control. This SMSr property is adapted to the involvement of two norms in the basic assumptions: smoothness, constraint qualification, and strong second order sufficient optimality conditions. The proofs are based on a new abstract result for strong metric subregularity (in a two-norms setting) of the Karush-Kuhn-Tucker optimality mapping for a mathematical programming problem in a Banach space, also presented in the paper.

## On the application of the semismooth* Newton method to variational inequalities of the second kind

**Jiří Outrata**

Czech Academy of Sciences, Institute of Information Theory and Automation

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The talk is devoted to the SCD (subspace containing derivative) variant of the semismooth* Newton method, proposed originally for the solution of general inclusions. After the description, this new method is applied to a class of variational inequalities of the second kind. As a result, one obtains an implementable algorithm exhibiting a locally superlinear convergence. Finally, this algorithm is used to compute Cournot-Nash equilibria, modeled as a variational inequality of the second kind.

## Rational activation functions in neural networks with uniform based loss functions and its applications in classification

**Vinesha Peiris**
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When the activation function of a simple neural network is one degree rational function and the loss function is based on the uniform norm, the overall optimisation problem of the network forms a generalised rational approximation problem. Moreover, when the coefficients of the rational activation function are fixed, the weights and the bias of the network become the decision variables of the corresponding optimisation problem. To optimise the decision variables, we suggest using two prominent methods: bisection method and differential correction algorithm. We perform numerical experiments on classification problems with two classes. We compare the classification accuracy obtained by the network using the bisection method, differential correction algorithm along with the standard MATLAB toolbox which uses least square loss function. We show that the choice of the uniform norm based loss function with rational activation functions and bisection method lead to better classification accuracy.

## Perturbed Fenchel Duality and First-Order Methods

**Javier Peña**
Carnegie Mellon University
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We show that the iterates generated by a generic first-order meta-algorithm satisfy a canonical perturbed Fenchel duality inequality. The latter inequality in turn readily yields a unified derivation of the best known convergence rates for various popular first-order algorithms including the conditional gradient method as well as the main kinds of Bregman proximal methods: subgradient, gradient, fast gradient, and universal gradient methods.

This is joint work with David H. Gutman at Texas Tech University.

## Derivative-Free Optimization with Convex Constraints

**Lindon Roberts**
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When optimizing functions which are computationally expensive and/or noisy, gradient information is often impractical to obtain or inaccurate. As a result, so-called "derivative-free" optimization (DFO) methods are a suitable alternative. In this talk, I will show how existing methods for interpolation-based DFO can be extended to nonconvex problems with convex constraints, accessed only through projections. I will introduce a worst-case complexity analysis and show how existing geometric considerations of model accuracy (from the unconstrained setting) can be generalized to the constrained case. I will then show numerical results in the case of nonlinear least-squares optimization. This is joint work with Matthew Hough (University of Queensland and University of Waterloo).

## Semi-continuity of the Solution Maps of Set-valued Equilibrium Problems with Equilibrium Constraints

Pradeep Kumar Sharma
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In this talk, we present the parametric set-valued equilibrium problems with equilibrium constraints. We introduce new notions of generalized concavity for set-valued maps and study their properties as well as their relationship with other existing well-known notions. By using the generalized concavity and semi-continuity of set-valued maps, we establish sufficient conditions for lower/upper semi-continuity of the solution maps of the set-valued equilibrium problems involving set order relations. An application of our results to multi-criteria traffic network equilibrium problems will be given. Several examples are given to illustrate the derived results.

## Two mathematical memories among many others on our late colleague and friend Asen Dontchev

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In this talk, I intend to briefly report on two papers related to Asen's work. The first one will be related on a join paper with him and Samir Adly. The other one will be about one of his work with Vladimir Veliov on $\omega$–limit sets.

## Abstract Convexity on Duality theory of Composite Minimization Problem

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We study conjugate and Lagrange dualities for composite optimization problems within the framework of abstract convexity. We provide conditions for zero duality gap in conjugate duality. For Lagrange duality, intersection property is applied to obtain zero duality. Connection between Lagrange dual and conjugate dual is also established. Examples related to convex and weakly convex functions are given.

## Optimization in Data Science

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Optimization is vital to the modern revolution in data science. Techniques from optimization have become essential in formulating and solving a wide variety of problems in data analysis, machine learning, and AI. In turn, these areas have prompted a ferment of new research activity in optimization by posing challenging new problems and new contexts. We give a brief overview of the many problems in data science in which optimization provides the key solution methodology. We then focus on several areas of recent interest in the intersection of optimization and data science, in such areas as nonconvex optimization, robust optimization, neural networks, matrix optimization, and adversarial machine learning.

## Extra Anchored Gradient Method for Convex-Concave Minimax Problems

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Algorithmic approaches to minimax problems have recently been paid much attention, due to their important applications in machine learning, in particular, in generative adversarial nets (GANs). In this talk we will recall and discuss some traditional and recent extra gradient and extra anchored gradient methods for convex-concave minimax problems. A theoretical ingredient is Halpern's anchored iterative method for nonexpansive mappings, which has

recently been proved to play an important role in solving variational inequalities and minimax problems.

## Lipschitz-like property relative to a set and the generalized Mordukhovich criterion

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In this paper we will establish some necessary condition and sufficient condition respectively for a set-valued mapping to have the Lipschitz-like property relative to a closed set by employing regular normal cone and limiting normal cone of a restricted graph of the set-valued mapping. We will obtain a complete characterization for a set-valued mapping to have the Lipschitz-property relative to a closed and convex set by virtue of the projection of the coderivative onto a tangent cone. Furthermore, by introducing a projectional coderivative of set-valued mappings, we establish a verifiable generalized Mordukhovich criterion for the Lipschitz-like property relative to a closed and convex set. We will study the representation of the graphical modulus of a set-valued mapping relative to a closed and convex set by using the outer norm of the corresponding projectional coderivative value. For an extended real-valued function, we will apply the obtained results to investigate its Lipschitz continuity relative to a closed and convex set and the Lipschitz-like property of a level-set mapping relative to a half line.